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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

SOLUTİON OF OUTPUT STABİLİZATİON PROBLEMS BY TİME AND FREQUENCY CALCULATİON METHODS

Specialty: 3338.01– System analysis, control and information processing (on optimal control)

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GENERAL CHARACTERISTICS OF THE WORK

Rationale and development degree of the topic.

Theory of optimal control is one the leading fields of modern mathematics. It is connected with the solution of large spectrum of rather complex mathematical problems and has important applications in many fields of science and engineering. This belongs on one hand to optimal stabilization problems of technological modes characterized by important mathematical compex, on the other hand, by many-dimensional linear objects encountered in practice.

The problem of optimal stabilization of any process occupies an important place in the row of optimal control problems. Such stabilization problems cover a wide range. It should be noted that the study of optimal stabilization problems is associated with classic methods of stability theory. In special case, by its essence, one of the main problems of optimal control theory, the dynamic programming method is the combination of variational calculus methods and the method of Lyapunov function. Improvement of the Lyapunov method has enabled to find efficient for methods for the solution of optimal stabilization problems. In their works, Larin V.B., Aliyev F.A., Veliyeva N.I. and others have obtained some important results for optimal stabilization of the motion of stationary systems. Control and optimization problems, in creating modern technology, for example control of chemical processes, oil production by gas lifting and socker rod pipe (maximum energy production, maximum row material, etc.) robots systems, control of spaceships, etc. play an important role. In spite of many theoretical researches with linear and nonlinear control problems in this field, great attention was paid to development of numerical algorithms for creating new technologies.

Development of a method or an algorithm for solving optimization and control problems does not mean that they can solve some practical problems i.e. to verify if they work, appropriate programs, for example, MATLAB MATHEMATICS and other mathematical program packet should be used. It is know that it is very difficult to build analytic solution of optimal control problems.

Therefore, a special attention should be paid to different approximate and numerical methods for solving them. According to features of problems, one of the suggested algorithm is applied. The solution of the problem of statsitical stabilization with respect to output is reduced to the solution of two nonlinear matrix equations and it is very difficult to solve these equations. The solution of these equations is still being researched. It is connected with the fact that as problms arise, the demand for these equations is increasing. For example, recently the optimal stabilization problems related to the movement of unmenned aerial vehicles reduces to the solution of the Riccati and Silvester equation. Although these equations were studied a lot, some shortcomings remain. These works are relevant in terms of application and have a great importance.

The dissertation work consists of introduction and three chapters.

In chapter I, solution methods for building optimal stabilizers with respect to periodic output for the continuous and discrete case, are suggested. In section 1 of chapter I stabilization problems where the motion of the subject is described by finite difference relations and by the system of differential equations in different parts of time, were researched and a solution algorithm was given.

In section 2 solution methods were developed for the discrete case of the problem of periodic optimal stabilization with respect to output.

In section 3, creation of stabilization algorithm for periodically controlled systems (continuous and discrete) within the feedback (a system for assessing spartial coordinates of a control object) was studied. Algorithms influencing on the character of transition process that occurs in the filter by means of some matrices characterizing intensity of sound and perturbation acting on the object are given and appropriate software was created.

In chapter II of the dissertation work, an optimization problem with a non-separated three point boundary condition was considered. Some methods, including the sweep method were offered. Application of the solution of this problem in construction of optimal stabilization in operation of oil wells by the gas-lift method, was studied.

In section 1 of chapter II an optimization problem with a non-separated boundary condition at inner and end points was considered, a sweep method for solving the appropriate continuous problem was offered.

In section 3, using the constructed mathematical model and appliying the straightline method, we obtain a quasiquadratic optimal control problem and unlike the initial problem, we give a solution algorithm by applying the known solution methods of an optimal control problem.

In chapter III for solving a problem on building output optimal stabilizers the methods for solving algebraic Riccati, Silverster equations are given.

For solving the normal state discrete BHH (Bevis-Hall-Hartwig) equation, an algorithm based on calculation of linear matrix inequalities in MATLAB medium is offered. The obtained result is analyzed on an example and the efficiency of the offered method is shown.

Unlike the reduction of BHH matrix equation to the classic Stein equation in section 1, a solution algorithm based on linear matrix inequalities (LMI) is given.

It is shown that the LMI algorithm offered for solving the BHH equation compared to known method is implemented more easily and conveniently. The results are given on examples. In section 2 the solution of the Riccati equation is researched in the general case, when the stabilizer i.e. all given values of the matrix of the closed system are located inside the a unit radius circle and when the anti stabilizer, i.e. all eigen values of the matrix of the closed system is outside a unit radius circle. The fast iteration scheme of the Riccati equation is given by the finite power series both in the continuous and discrete case. The iterative scheme was verified by the examples covering different cases.

Object and subject of the study. The object and subject of the study are problems of stabilization with respect to output, optimization problems with non-separated three point boundary condition at inner and end points.

Goals and objectives of the study. The goal of the dissertation work is to work out time and frequency calculation methods of a stabilization problem, to elaborate a solution algorithm for an optimization problem with non-separated three-point boundary conditions at the inner and end points and to apply the obtained results in oil production by the gas-lift method.

To achieve the indicated goal, the solution of the following problems was offered:

- A method for solving a problem of periodic optimal stabilization with respect to output in the continuous case;
- A method for solving a problem of periodic optimal stabilization with respect to output in the discrete case;
- Iterative solution method for a problem of periodic optimal stabilization with respect to output in the discrete case;
- A sweep method for solving an optimization problem with non-separated three-point boundary condtion at inner and end points;
- Application of the solution of optimal control problem with three-point boundary condition by the gas-lift method;
- The method for solving the BHH Silvester equation;
- A method for solving the Riccati equation.

Research methods. Optimization methods, numerical methods, theory of differential equations, gas-lift method were used for solving the stated problems.

Implementation and application of the results of the work.

The main results of the work were used in executing grant projects on operation of oil by efficient methods.

The main theses to be defended. The scientific novelties of the research and obtained results are the followings:

✓ Working out a method for solving output periodic optimal stabilization problem in the continuous case ;

✓ Working out a method for solving output periodic optimal stabilization problem in the discrete case;

✓ Working out iterative solution method of output periodic optimal discretezation problem in the discrete case;

✓ Working out a sweep method for solving an optimization problem with non-separated three-point boundary condition at inner and end points;

✓ Application of three-point boundary condition optimal control problem to oil production by gas-lift method;

✓ Working out a method for solving the BHH Silvecter equation;

✓ Working out the Riccati equation by the fast iterative solution methods.

Scientific novelty of the study and the theses to be defended.

A method for solving output periodic optimal stabilization problem in the discrete case was developed;

An iterative mnethod for solving output periodic optimal stabilization problem in the discrete case, was given;

A sweep method for solving non-separated three-point boundary condition optimization problem at inner and end points was elaborated;

The solution of three-point boundary condition optimal control problem was applied to oil production by gas-lift method;

A method for solving the BHH Silvester equation was worked out;

A method for solving the Riccatu equation was developed.

Theoretical and practical importance of the dissertation work.

Practical importance of the work is that its scientific-theoretical results can be applied in other fields of science including in oil production industry.

Approbation and application. Scientific theoretical and practical results of the work were reported and discussed at the following scientific conferences:

– The 5th International Conference on Control and Optimization with Industrial Applications, 27-29 August, 2015, Baku, Azerbaijan;

– Scientific seminars of scientific-research Institute of Applied Mathematics of Baku State UNiversity;

– The VI congress of the TWMS, October 2-5, 2017, Astana, Kazakhstan;

– Proceedings of the 6th International Conference on Control and Optimization with Industrial Applications (COIA 2018);

– Proceedings of the 7th International Conference on Control and Optimization with Industrial Applications (COIA 2020).

Author's personal contribution. The obtained results and statements belong to the author.

Author's publications. The main results of the work were published in 11 scientific papers the list of which is at the end of the abstract.

The name of organization where the dissertation work was performed. The work was performed at the Scientifi research Institute of Applied Mathematics of Baku State University.

Total volume of the dissertation work indicating separately the volume of each structural unit in signs:

The total volume of the dissertation work consists of-210360 signs (title page – 470 signs, contents – 960 signs, introduction – 17931 signs, chapter I -68000 signs, chapter II -78000 signs, chapter III-44000 signs, conclusion-822 signs). The list of ised references with 152 names.

CONTENT OF THE WORK

In the introduction the rationale of the work was commented, the bases of the conducted work were underlined, theoretical and practical importance of the work was reflected, the theses to be defended were indicated, the content and structure of the work, the desired conclusions to be defended were described.

Chapter I was devoted to the method for solving an output periodic optimal stabilization problem.

In section 1 of chapter I the solution of the stabilization problem where the motion of the object is described by both finite-difference relations and by the system of differential equations at different parts of time was considered.

Let in the interval $(k-1)\tau < t < k\tau,\ k = 1,2,\ldots,$ the motion of the control object be expressed by the following system of differential equations

$$\dot{x} = Ax + Bu \tag{1.1.1}$$

at the moment $t = k\tau$ be described by the finite difference relations

$$x(k\tau + 0) = Nx(k\tau - 0) + Mv(k) \tag{1.1.2}$$

It is required to find such continuous and impulsive control strategy that the object +regulator closed system

$$u(t) = f(x(t)), v(k) = \varphi(x(k\tau - 0)) \quad ,$$

be asymptotically stable and afford minimum (quality criterion) value to the following quadratic functional:

$$I(t_0) = \int_{t_0}^{\infty} (x'Qx + u'Ru)dt + \sum_{k=1}^{\infty} v'(k)Cv(k) \ . \tag{1.1.3}$$

Here the matrices $A, B, R = R' > 0,\ Q = Q' \geq 0$-are of period τ with respect to the variable t the matrices $N, M, C = C' > 0$ are constant x, u are spartial coordinates of appropriate dimension and are the vectors of control action.

Using the solution of the linear Quadratic Gauss method, we find the minimum of the functional (1.1.3) by using the method of finding the quadratic form

$$\min_{u,v} I(t_0) = x'(t_0)S(t_0)x(t_0) .$$

here $\tau < t < k\tau$ in the interval $(k-1)$ is found by the formula

$$u = -R^{-1}B'Sx, \tag{1.1.4}$$

in the interval $t = k\tau$ by the formula

$$v(k) = -(M'S(k\tau + 0)M + C)^{-1} M'S(k\tau + 0)Nx(k\tau - 0). \tag{1.1.5}$$

The matrix S is found in the interval $(k-1)$ $\tau < t < k\tau$ from the following differential Riccati equation

$$-\dot{S} = SA + A'S - SBR^{-1}B'S + Q . \tag{1.1.6}$$

The jump of this matrix at the moment $t = k\tau$ is expressed by the following expression

$$S(k\tau - 0) = N'\{ (S(k\tau + 0) - S(k\tau + 0)M\left(C + M'S(k\tau + 0)M\right)^{-1} \times \tag{1.1.7}$$
$$\times M'S(k\tau + 0) \}N.$$

So, to determine optimal strategy of control we have to find such a periodic (of τ period) matrix S that the equations (1.1.6) , (1.1.7) be satisfied and the systems (1.1.1), (1.1.2), (1.1.4), (1.1.5) be asymptotically stable. An algorithm for solving this problem was suggested.

In sections 2 of chapter I a method for solving a periodic optimal stabilization problem in the discrete case was considered and an algorithm was offered. Let us study a problem on analytic contruction of stabilizers for a discrete periodic system.

Assume that the motion of the object was given by the system of finite differences equation

$$x(i+1)=\psi(i)x(i)+\Gamma(i)u(i), i=0,1,2,... \qquad (1.2.1)$$

Choosing appropriate control (regulator equation) strategy

$$u(i)=f(x(i)) \qquad (1.2.2)$$

within the condition $x(0)\neq 0$ it is required to provide the stability of the system (1.2.1) and (1.2.2) ($\lim_{i\to\infty}(i)=0$) and the quadratic functional

$$I=\sum_{i=0}^{\infty}\left[x'(i)Q(i)x(i)+u'(i)R(i)u(i)\right]. \qquad (1.2.3)$$

take a minimum value.

Here $x(i),u(i)$ are spartial coordinates and the vectors of control action, $\psi(i), \Gamma(i), Q(i)=Q'(i)\geq 0, R(i)=R'(i)>0$ $p,$ are periodic matrices, i.e. the periodicity conditions $\psi(i+p)=\psi(i),\ \Gamma(i+p)=\Gamma(i),$ are satisfied.

It is known that the control of optimal regulator (1.2.2) is as follows :

$$u(i)=-\left[\Gamma'(i)S(i+1)\Gamma(i)+R(i)\right]^{-1}\Gamma'(i)S(i+1)\psi(i)x(i). \qquad (1.2.4)$$

The sequence of symmetric S matrices in optimal control law is defined from the following recurrent relation

$$S(i)=\psi'(i)[S(i+1)-S(i+1)\Gamma(i)+R(i)+\Gamma'(i)S(i+1)\Gamma(i))^{-1}\Gamma'(i)S(i+1)\psi(i)+Q(i)\ ]. \qquad (1.2.5)$$

So, to define the control law (1.2.4) it is necessary to find one of the sequences of matrices $S(i)$ satisfying the condition (1.2.5).

As the matrices in the condition of the problem are periodic, shifting the initial moment of the process to the index p $(i = p, p+1, \ldots$) the strategy of the control will not chage. Therefore, the sequence of desired matrices should satisfy the periodicity condition $S(i+p) = S(i)$ After making some transfunctions

$$\psi(i,n+1) = \psi(i+n)(E + D(i,n)Q(i+n))^{-1}\psi(i,n),$$
$$\psi(i,0) = E,$$
$$D(i,n+1) = \psi(n+i)[(E + D(i,n)Q(i+n))]^{-1} D(i,n)\psi'(n+i) +$$
$$+ \Gamma(n+i)R^{-1}(n+i)\Gamma'(n+i),$$
$$D(i,0) = 0,$$
$$Q(i,n+1) = Q(i,n) + \psi'(i,n) + Q(n+i)[\ E + D(i,n)Q(i+n)\ ]^{-1}\psi(i,n),$$
$$Q(i,0) = 0.$$

the discrete algebraic Riccati equation will take the following form

$$S(i) = \psi'(i,p)\{S(i) - S(i)D(i,p)[(D(i,p))'S(i)D(i,p) + +(U(i,n))^{-1}\ ]^{-1}(D(i,p))' S(i)\ \}\psi(i,p) + Q(i,p). \tag{1.2.6}$$

Theorem. If the matrix (1.2.6)

$$[E + D(i,p)S(p+i)]^{-1}(\psi(i,p)),$$

that determines the change of vector space of the closed system object-regulator has a solution whose eigen values are inside a unit radius circle, then the system of equations (1.2.1), (1.2.4) asymptotically stable and these values $S(i)$ express the desired periodic sequence.

In section 3 of this chapter the construction of stabilizarion algorithm of periodic systems within feedback (the system for estimation of spatial coordinates of the controlled object) is shown for a linearly periodically controlled (continuous and discrete) system.

In section 4 of chapter I an algorithm for solving the output linear-quadratic periodic optimal regulator feedback problem. The solution of such problems was considered in the papers of Bittanti S.H., Levine W.S., Athans M., Aliev F.A., Safarova N.A. In the papers of Peres P.L.D., Geromel J.C. a convex programming device, in the papers of Aliev F.A., Larin V.B the associated gradients method are used. In these works, each time the Lyapunov equation is solved and sometimes this can negatively effect on the exactness of the solution. In this section an iteration algorithm for solving the output optimal regulator problem is applied and it is not reguired to solve Lyapunov matrix algebraic equations at each step.

Cahepter II. In this chapter we consider a three-point boundary condition optimization problem not separated at inner and end points. To determine the boundary conditions that miss because of non-ctandard property of boundary conditions, we use initial data of Lagrange polynomial.

In section 2 of chapter II a sweep method for solving appropriate continuous problem is offered, i.e. the solution of the Riccati matrix differential equations and also the solution of linear matrix differential equations are used.

This method differs from other ones with the fact that in this approach we need not to increase the dimension of the initial system.

It is known that one of the main methods for finding program trajectories in oil production (Aliev F.A., Ilyasov M.Kh., Jamalbekov M.A.) and in operation of robots (Larin V.B.) is the use of the solution of three-point boundary condition optimization problem. Usually, in many cases, boundary conditions are given in not-completely separated form or at some points separated boundary conditions are given. Such problems occur in oil production by the gas-lift method or when chosing the volume of gas injected to extract gas-fluid mixture.

The case when the initial data of the boundary conditions are complete and not separated at inner and end points are especially of interest.

Assume that on the interval $[0,\tau)$, $(\tau,T]$ the equation of motion of the object is described by controlled nonstationary system of linear differential equations

$$\dot{x} = Fx + Gu + v \qquad (2.1.1)$$

here the solution should satisfy the initial condition

$$x(0) = x^0 \ , \tag{2.1.2}$$

and the points τ, T the coordinates $x(\tau),\ x(T)$ should satisfy the not separated boundary conditions

$$Ax\ (\tau) = Bx(T)\,. \tag{2.1.3}$$

Let us construct a quadratic functional as follows:

$$J = \frac{1}{2} x'(T) S_f\, x(T) + \frac{1}{2}\int_0^T [x'(t)Rx(t) + u'(t)Cu(t)]dt\,, \tag{2.1.4}$$

here $S_f = S_f' < 0,\ R = R' > 0,\ C = C' > 0$, are the given matrices of appropriate dimension, the sign of prime indicates the transposition operation.

So, it is reguired to find the solution of the problem, (2.1.1) - (2.1.3) that affords a minimum value to the functional (2.1.4).

Now, let us express the solution algorithm for the problem (2.1.1)-(2.1.4).

1. We are given the matrix's $F,\ G, R,\ C, S_f,\ A, B$ and the vectors $\nu(t), x^0$.

2. The Cauchy problem is solved and on the interval $(\tau + 0, T]$ the functions $S(t), N(t), \omega(t)$ are found.

3. From $\begin{cases} S(\tau + 0) = S(\tau - 0) \\ N(\tau + 0) = N(\tau - 0) - A' \\ \omega(\tau + 0) = \omega(\tau - 0) \end{cases}$

$S(\tau - 0), N(\tau - 0), \omega(\tau - 0)$ are determined. Using these initial values on the interval $(0, \tau - 0]$ we find the functions $S(t), N(t), \omega(t)$.

4. Solving the equation $\begin{cases} \dot{n}(t) = N'(t)M(t)N(t), \quad n(T)=0 \\ \dot{W}(t) = N'(t)[M\omega(t) - v(t)], \, W(T)=0 \end{cases}$ within the conditions $n(t), W(t)$ we find the functions $n(\tau-0)=0, W(\tau-0)$ on the interval $(0,\tau), \tau+0, T)$.

5. We determine $\lambda(0), x(\tau), \gamma$ from the system of algebraic equations.

6. Solving the equation

$$\dot{x}(t) = \left(F(t) - M(t)S(t)\right)x(t) - M(t)N(t)\gamma + \left(v(t) - M(t)\omega(t)\right).$$

within the initial conditions $x(0)$ we find the function $x(t)$.

7. According to the formula

$$u(t) = -C^{-1}(t)G'(t)S(t)x(t) - C^{-1}(t)G'(t)N(t)\gamma - C^{-1}(t)G'(t)\omega(t)$$

we determine the desired control .

In section 2 of chapter II we offer a solution algorithm for a discrete optimal control problem with not separated three-point boundary conditions at inner and end points.

It is shown that such boundary value problems can be applied to some practical problems including operation of oil wells by the gas-lift method.

If the control u is a piecewise constant function and F, G, υ, R, C are constant matrices, the continuous optimal control problem, (2.1.1)-(2.1.4) can be easily reduced to the discrete problem

$$x_{i+1} = \psi_i x_i + \Gamma_i u_i + v_i, \;\; i = 0,1,...,s-1, \; s, s+1,...,l-1 \qquad (2.2.1)$$

$$x_0 = \overline{x} \qquad (2.2.2)$$

$$Ax_s = Bx_l \qquad (2.2.3)$$

here the matrices ψ_i, Γ_i, v_i are determined as follows

$$\psi_i = e^{F\Delta}, \quad v_i = F^{-1}(e^{F\Delta} - E)\upsilon, \quad \Gamma_i = (\int_0^{\Delta} e^{F\xi} d\xi)G.$$

$$J=\frac{1}{2}x_l'S_f x_l+\frac{1}{2}\sum_{i=0}^{l-1}\left[x_i'R_i x_i+u_l'C_i u_i\right] \qquad (2.2.4)$$

As in the continuous case, the solution of the problem (2.2.1-2.2.4) can be found in a similar way.

In this case:

$$\lambda(T)=S(T)x(T)+N(T)\gamma+\omega(T)$$

is found in the form of

$$\dot{x}(t)=\left(F(t)-M(t)S(t)\right)x(t)-M(t)N(t)\gamma+\left(\nu(t)-M(t)\omega(t)\right).$$

and S_S, N_S ω is found from the relations

$$S_s=R_s+\psi_s{}'S_{s+1}\left(E+M_sS_{s+1}\right)^{-1}\psi_s \qquad (2.2.5)$$

$$N_s=A'+\psi_s{}'\left[E-S_{s+1}\left(E+M_sS_{s+1}\right)^{-1}M_s\right]N_{s+1} \qquad (2.2.6)$$

$$\omega_s=\psi_s{}'\left[E-S_{s+1}\left(E+M_sS_{s+1}\right)^{-1}M_s\right]\omega_{s+1}+\psi_s{}'\omega_{s+1}\left(E+M_sS_{s+1}\right)\nu_s \qquad (2.2.7)$$

In section 3 of chapter II the solution algorithm of a three-point boundary condition optimal control problem and its application to oil production by the gas-lift method, was studied. In this section, an optimal control problem with a boundary condition with not separated inner and end points was studied and statement of boundary conditions in this form is connected with the existence of some concrete problems, including applications of the gas-lift method in oil production.

When operating the well by the gas-lift method, an calculated algorithm to study and control its operation mode was prepared. To state an optimal control problem corresponding to the gas-lift process it is necessary to built a mathematical model of operation of wells by the gas-lift model.

One of the problems of the gas-lift process is complete production of gas-lift mixture formed in the botton of the lift, in the form of output. Practice shows that only 38% of gas-fluid mixture is extracted from the well in the form of output. To get maximum output by injecting minimum gas volume it is offered that the volume of the gas-fluid mixture in front of and at the end of the lift be equal, (i.e. the periodicity condition should be fuldilled) this means that QMQ is completely transmitted from the mixture zone to the well.

In this section, using the built model and the straightline method we obtain a linear quadratic optimal control problem and unlike the initial problem we can apply the known solution methods of optimal control problem to this problem. The straightline method is based on the assumption that the oil-well tubing consists of finitely many pieces of certain lengt hand in each on them one can get ordinary differential equations. Here the injected gas is taken as a control parameter, the injected gas and is functional dependent on the output of the well is taken as a minimized functional.

Thus, the optimal control problem consists of minimization of the injected gas volume anbd obtaining maximum value of the well output.

When operating the wells by the gas-lift method, gas is continuously injected. For bubbly fluid gas structures the model of the operation of the gas-lift well is approximately described by the followoing system of partial differential equations, i.e. the gas-lift process is expressed by the system

$$\begin{cases} \dfrac{\partial P}{\partial t} = -\dfrac{c^2}{F}\dfrac{\partial Q}{\partial x} & t \geq 0, \\ \dfrac{\partial Q}{\partial t} = -F\dfrac{\partial P}{\partial x} - 2aQ & x \in [0,2l] \end{cases} \tag{2.3.1}$$

here $t \geq 0,\ x \in [0,2L]$, L is the well depth, F is the area of cross-section of piping along the axis x, c is the light speed in gas or gas-fluid (GFM) mixture, a is a hydraulic resistance, P and Q is the excess pressure velocity of change of the fluid volume.

Note that in the equation (2.3.1) the coefficients F, c, a are determined as follows:

$$F = \begin{cases} F_1 & x \in (0, L) \\ F_2 & x \in [L, 2L] \end{cases};\quad c = \begin{cases} c_1 & x \in (0, L) \\ c_2 & x \in [L, 2L] \end{cases};\ a = \begin{cases} a_1 & x \in [0, L) \\ a_2 & x \in [L, 2L] \end{cases}$$

Assume that a pipe of length L consists of N number pipes of the length l $\left(k=\overline{1,N}\right)$. If in each segment we accept the relation

$$\frac{\partial P}{\partial x} \approx \frac{P_k - P_{k-1}}{l} \text{ və } \frac{\partial Q}{\partial x} \approx \frac{Q_k - Q_{k-1}}{l}, \; l = \frac{L}{N}, \; k = \overline{1,2N}$$

Then we can write the system (2.3.1) in the form of ordinary differential equations

$$\begin{aligned} \frac{dP_k}{dt} &= -\frac{c^2}{Fl}(Q_k - Q_{k-1}) \\ \frac{dQ_k}{dt} &= -\frac{F}{l}(P_k - P_{k-1}) - 2aQ_k \end{aligned} \qquad k = \overline{l,2N}\,.$$

(2.3.2)

The volume and pressure consumed in the lift bottom can be shown as follows

$$\tilde{P}_N = P_n + P_{Pl}\;, \quad \tilde{Q}_N = Q_n + Q_{Pl} \tag{2.3.3}$$

Taking into account the expressions (2.3.3) in the equations (2.3.2) we obtain the following equations characterizing this process in the lift, i.e. in the domain corresponding to the part $k = \overline{N+1,2N}$

$$P_k(t_0) = P_k^0, \;\; Q_k(t_0) = Q_k^0 \qquad k = \overline{0,2N} \tag{2.3.4}$$

Thus, the get the Cauchy problem for the system of first order ordinary differential equations.

We can show the problem (2.3.2)-(2.3.4) in the following matrix form:

$$\dot{x} = Fx + Gu + \vartheta\,, \;\; x(0) = x^0\,, \tag{2.3.5}$$

$$Q_N(\tau) = Q_{2N}(T)\,, \quad \tau < T \tag{2.3.6}$$

In fact, the conditions (2.3.6) provide compression of gas-fluid mixture without loss in the lift. Denote

$$x = [P_1, Q_1, ..., P_N, Q_N, P_{N+1}, Q_{N+1}, ..., P_{2N}, Q_{2N}]'$$

$$x^0 = [P^0{}_1, Q_1{}^0, ..., P_N{}^0, Q_N{}^0, P^0{}_{N+1}, Q^0{}_{N+1}, ..., P^0{}_{2N}, Q^0{}_{2N}]'\,.$$

It is reguired to find a solution of the problem (2.3.1), (2.3.4) that affords a minimum to the functional

$$J=\frac{1}{2}x'(\tau)S_f x(\tau)+\frac{1}{2}\int_0^T\left[\,x'(t)Rx(t)+\ u'(t)Cu(t)\right]dt$$

We can offer the following algorithm:

1. We are given x^0, $c, L, a, R, C, S_f, T, \tau$.

2. The functional G, F, υ are determined.

3. We find $H=\begin{bmatrix} F & -GC^{-1}G' \\ -R & -F' \end{bmatrix}$.

4. The matrices $\begin{bmatrix} H_{11}^1 & H_{12}^1 \\ H_{21}^1 & H_{22}^1 \end{bmatrix}$ və $\begin{bmatrix} H_{11}^2 & H_{12}^2 \\ H_{21}^{'2} & H_{22}^{'2} \end{bmatrix}$ are formed:

$$e^{H\tau}=\begin{bmatrix} H_{11}^1 & H_{12}^1 \\ H_{21}^1 & H_{22}^1 \end{bmatrix}, \qquad e^{H(T-\tau)}=\begin{bmatrix} H_{11}^2 & H_{12}^2 \\ H_{21}^{'2} & H_{22}^{'2} \end{bmatrix},$$

5. The vector e and the matrix M are formed:

$$M=\begin{bmatrix} 0 & 0 & -E & E & 0 & 0 & A' \\ 0 & 0 & 0 & 0 & -S_f & -E & B' \\ 0 & A & 0 & 0 & -B & 0 & 0 \\ -H_{12}^1 & E & 0 & 0 & 0 & 0 & 0 \\ -H_{22}^1 & 0 & E & 0 & 0 & 0 & 0 \\ 0 & -H_{11}^2 & 0 & -H_{12}^2 & E & 0 & 0 \\ 0 & -H_{21}^2 & 0 & -H_{22}^2 & E & 0 & 0 \end{bmatrix}$$

6. Solving the equation $Mz=e$ we z find the vector:

$$z'=[\lambda(0), x(\tau), \lambda(\tau-0), \lambda(\tau+0), x(T), \lambda(T), \nu]$$

7. We find the quantities $x(0)$, $\lambda(0)$ of the vector z .

8. The system of differential equations $\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} F & -GC^{-1}G' \\ -R & -F' \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix};$ is solved in the interval $[0 \quad \tau)$, $(\tau \; T)$ and the functions $x(t), \lambda(t)$ are determined.

9. Control actions u, $x(\tau), \lambda(0)$ are found from the formula $u = -C^{-1}G'\lambda,$.

In chapter III an algorithm based on the process of calculation of linear matrix inequalities in MATLAB medium is offered to solve the discrete BHH equation. At the same time, solution algorithm of the algebraic Riccati equation is given. The result is analyzed on an example and the efficiency of the offered method is shown.

In section 1 of chapter III unlike the reduction of the BHH (Bevis–Hall–Hartwig) matrix equation to the classic Stein equation $X - A\overline{X}B = C$ a solution algorithm bvased on the linear matrix inequalities (LMI) is given. It is shown that the LMI algorithm offered to solve the BHH equation is implemented simply and conveniently than the known methods. The results are shown on examples.

Let us consider the BHH Silvester equation.

$$X - A\overline{X}B = C \qquad (3.1.1)$$

Reducing the equation (3.1.1) to the Stein equation

$$X - (A\overline{A})X(\overline{B}\;B) = C + A\overline{C}\,B \qquad (3.1.2)$$

İt can be solved by applying the standard dyap m procedure of MATLAB system.

Here the coefficients A,B are adjoint normal matrices i.e.

$$AA^* = A^*A, \;\; BB^* = B^*B \; .$$

The matrices A and B are $m \times m$ and $n \times n$ dimensional, respectively, the desired C and X matrices are $m \times n$ dimensional.

Applying the LMI algorithm, we can annihilate the procedure (3.1.2) and directly solve equation (3.1.1).

At first assume that in the equation (3.1.1) A, B and C are complex matrices and are expressed in the form of

$$A = A_1 + iA_2,\ \ B = B_1 + iB_2,\ C = C_1 + iC_2,\ \ X = X_1 + iX_2$$

Then from equality of real and imaginary part of the equation (3.1.1) we obtain the following algebraic equations

$$\begin{aligned} X_1 - A_1 X_1 B_1 - A_1 X_2 B_2 + A_2 X_1 B_2 - A_2 X_2 B_1 &= C_1 \\ X_2 - A_1 X_1 B_2 + A_2 X_2 B_1 - A_2 X_1 B_1 - A_2 X_2 B_2 &= C_2 \end{aligned} \qquad (3.1.3)$$

$$\begin{gathered} n_1 = X_1 - A_1 X_1 B_1 - A_1 X_2 B_2 + A_2 X_1 B_2 - A_2 X_2 B_1, \\ n_2 = X_2 - A_1 X_1 B_2 + A_2 X_2 B_1 - A_2 X_1 B_1 - A_2 X_2 B_2, \\ K_1 = n_1 - C_1, \\ K_2 = n_2 - C_2, \end{gathered} \qquad (3.1.4)$$

Denoting $\begin{bmatrix} \Pi_i & K_i \\ K_i' & I \end{bmatrix} > 0,\ \ i = 1, 2, ..., k,\ \ \Pi_i = \Pi_i'$ according to the matrix inequality we can find the solution of the equation (3.1.1) from the following inequality

$$X_1 \geq 0,\ X_2 > 0,\ Y > 0$$

$$T_1 = C_2 - X_2 + A_1 X_2 B_1 - A_2 X_1 B_1 - A_1 X_1 B_2 - A_2 X_2 B_2$$

$$T_2 = C_1 - X_1 - A_1 X_1 B_1 - A_2 X_2 B_1 + A_2 X_1 B_2 - A_1 X_2 B_1$$

In section 2 of chapter III an effective method for solving the algebraic Riccati equation in discrete and continuous case is offered. At the same time, the stabilizing and anti-stabilizing solutions of the Riccati equation were studied by the method of infinite power series. The solution algorithm was verified by many examples in the MATLAB mathematical programs packet and a software was created.

Let us consider an algebraic continuous Riccati equation

$$SF + F'S - SGC^{-1}G'S + R = 0 \qquad (3.2.1)$$

here F, G are $n \times n$ and $m \times m$ dimensional matrices, respectively, and the pair of matrices (F, G) is a stabilizer. $R = R' \geq 0$, $C = C' > 0$ are $n \times n$ and $m \times m$ dimensional square matrices, respectively, ($F', R^{1/2}$) is a detected pair. To find the solution of non-negative $S = S' \geq 0$ (the eigen-values $F - GC^{-1}G'S$) are located in the left half-plane, we use the following method.

Using the Newton-Raphson algorithm, we apply the iterative solution method to the equation (3.2.1). The equation (3.2.1) is solved by reducing to the solution of the Liapunov equation making some transformations. For the first time the solution of the following Liapunov equation was considered

$$F'S + SF = -Q \tag{3.2.2}$$

In the Liapunov equation F and Q are the given contant matrices, $S = S'$ is a desired matrix satisfying the symmetricity condition. To solve this equation an infinite series method was applied. This is mainly is applied when the dimension of the matrix is great and eigen-values of the matrix F are located on the left half-plane.

Accept the denotations $u = (E - F)^{-1};\ \psi = u(E + F)$ and $R = 2u'Qu$, then the equation (3.2.2) is reduced to the discrete Liapunov equation

$$S - \psi' S \psi = Q \tag{3.2.3}$$

If in the equation (3.2.2) the eigen values of the matrix F are located in the left half-plane, then eigen-values of the matrix ψ in (3.2.3) are located inside a unit radius circle. The series

$$Y = R + (\psi')R\psi + (\psi')^2 R\psi^2 + (\psi')^3 R\psi^3 + \ldots \tag{3.2.4}$$

converges and is the solution of the equation (3.2.3) .
For that the iteration scheme is built as follows.

$$Y_0 = R$$

$$Y_{k+1} = Y_k + (\psi')^{2k} Y_k \psi^{2k} \tag{3.2.5}$$

$$Y_k = \sum_{i=1}^{2k} (\psi')^{i-1} R \psi^{i-1},\ k = 0,1,\ldots$$

İt is seen from the iteration scheme that with increasing the value of k the order of summable terms of the series doubly increases. The iterative scheme of the equation (3.2.3)

$$S_i = \psi' S_{i+1} \psi + Q \ . \tag{3.2.6}$$

Theorem. If in the iterative scheme (3.2.6) the characteristic numbers of the matrix ψ is located inside a unit cirle, then the solution S_i $i \to \infty$ of the equation (3.2.6) converges to the solution S for the values $i = 2; 2^2; 2^3; \ldots, 2^k$ of a unique solution of the equation (3.2.3).

Applying such a iteration scheme to the discrete algebraic Riccati equation, we build a solution scheme.

Main results

As a result of researches carried out in this dissertation work, we obtained the following results:

- ✓ A method for solving an output periodic optimal stabilization problem for a continuous case was offered and a calculation algorithm was worked out;
- ✓ A method for solving an output optimal periodic optimal stabilization problem for a discrete case was offered and on calculation algorithm was worked out;
- ✓ An iterative solution method for an output periodic optimal stabilization problem was elaborated;
- ✓ A sweep method for solving an optimization problem with nonseparated three-point boundary condition at inner and end points was developed and appropriate solution algorithm was worked out;
- ✓ The solution of three-point boundary condition optimal control problem was applied to the oil production by the gas-lift method and appropriate calculation algorithm was worked out;
- ✓ A method for solving the BHH Silvester equation was worked out;
- ✓ Fast iterative solution method for the Riccati equation was elaborated.

The main results of the dissertation work are in the following scientific publications:

1. Велиева Н.И., Сафарова Н.А., Фараджева Ш.А. Итеративный алгоритм для решения задачи оптимальной стабилизации дискретной периодической системы по выходу // Proceedings of IAM, 2014, V.3, N.2, с.196-204.
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3. Велиева Н.И., Муталлимов М.М., Фараджева Ш.А. Алгоритм решения задачи оптимального управления с трехточечными

граничными условиями с применением к добыче нефти газлифтным способом. // Proceedings of IAM, V.5, N.2, 2016, pp.143-155.
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10. Aliev F.A., Larin V.B., Velieva N.I., Gasimova K.G., Faradjova Sh.A. Algorithm for solving the systems of the generalized sylvester-transpose matrix equations using LMI.// TWMS J. Pure Appl. Math. V.10, N.2, 2019, pp.239-245.
11. Aliev F.A., Larin V.B., Velieva N.I., Gasimova K.G., Farajova Sh.A. LMI method for solving BHH equations.// Proceedings of the 7th International Conference on Control and Optimization with Industrial Applications (COIA 2020), Vol. 1, pp.95-97.

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